KAPITAŁ LUDZKI
NARODOWA STRATEGIA SPÓJNOŚCI
Projekt współfinansowany przez Unię Europejską w ramach Europejskiego Funduszu Społecznego
UNIA EUROPEJSKA
EUROPEJSKI FUNDUSZ SPOŁECZNY

Course title

Combinatorics

ECTS code

11.1.0324

Name of unit administrating study

null

Studies

faculty	field of study		
Faculty of Mathematics, Physics and Informatics	Mathematics	type	first tier studies (BA)
		form	full-time
		specialty	null, mathematics – teacher education
		specialization	all
Faculty of Mathematics, Physics and Informatics	Mathematics	type	second tier studies (MA)
		form	full-time
		specialty	theoretical mathematics, financial mathematics, mathematics – teacher education
		specialization	all
Faculty of Mathematics, Physics and Informatics	Mathematical Modeling and Data Analysis	type	second tier studies (MA)
		form	full-time
		specialty	all
		specialization	all

Teaching staff

prof. UG, dr hab. Andrzej Nowik; dr Marek Hałenda; dr Poj Lertchoosakul; dr Marta Frankowska

Forms of classes, the realization and number of hours

Forms of classes

Auditorium classes, Lecture

The realization of activities

classroom instruction

Number of hours

Lecture: 30 hours, Auditorium classes: 30 hours

ECTS credits

5

The academic cycle

2022/2023 summer semester

Type of course

an elective course

Language of instruction

- english
- polish

Teaching methods

- problem solving
- problem-focused lecture

Form and method of assessment and basic criteria for eveluation or examination requirements

Final evaluation

- Graded credit
- Examination

Assessment methods

- (mid-term / end-term) test
- written exam with open questions

The basic criteria for evaluation

Method of verifying required learning outcomes

Required courses and introductory requirements

A. Formal requirements

B. Prerequisites

Aims of education

The goal of this course is to present a selected notions and theorems from combinatorics.

Course contents

1. Fundamental techniques in discrete mathematics (counting the number of functions, permutations, subsets, etc.), Catalan numbers, inclusion-

exclusion principle.

2. Hall's marriage theorem and it's applications to Latin rectangles and tournaments (Landau's theorem). Bell numbers, Stirling numbers of the first and second kind and some relations between these numbers.
3. Latin squares and their basic properties.
4. Theorems concerning Latin square extensions. Recently solved problems about Latin squares extensions (Dinitz conjecture, Evans' conjecture).
5. Mutually orthogonal Latin squares. Definition of the number N(n) and its properties.
6. Ramsey's theorem (finite and infinite version). Definition of the Ramsey number. Well known bounds for the Ramsey numbers. Some examples of Ramsey numbers (R(3,3), R(3,4)).
7. Partition theorems: Hales–Jewett theorem, Van der Waerden’s theorem, Schur's theorem and sum-free sets, Szemeredi's theorem (without proof).
8. Matroids and greedy algorithms. Some unsolved problems in combinatorics, for example: the Frankl conjecture, Erdos conjecture on arithmetic progressions, etc. Open problems in Ramsey theory.

Bibliography of literature

,,Wstęp do matematyki dyskretnej", A. Szepietowski.
,,Kombinatoryka", W.Lipski.
,,Wykłady z kombinatoryki", Z. Palka, A. Ruciński.
"Combinatorics: Topics, Techniques, Algorithms", P. Cameron

The learning outcomes (for the field of study and specialization)

Knowledge

- The student knows the definitions and properties of basic combinatorial notions (counting of functions, permutations, subsets, the Catalan numbers, the Bell numbers,the Stirling numbers, etc.)
- Is familiar with major theorems from combinatorics (for example the inclusion–exclusion principle, Hall's marriage theorem, Ramsey theorem, et cetera.)
- The student knows the infinite version of the Hall's marriage theorem.
- The student can formulate at least one theorem about the approximation of the number of systems of distinct representatives.
- The student knows the definition of a Latin square, Graeco-Latin square and the definition of mutually orthogonal Latin squares.
- The student can formulate the thirty-six officers problem and knows the interpretation of this problem in the language of Graeco-Latin squares.
- The student knows the definition and basic properties of the number MOLS(n) and is able to formulate at least open question about this coefficient.
- The student knows for which numbers less than 10 there does not exist a Graeco-Latin square of size n x n.
- Knowledge of the formulations of both versions of the Ramsey theorem (finite and infinite version).
- Knowledge of the correct interpretation of the formulas R(n, k) <= M, R(n, k) >= M and R(n, k) = M.
- The student knows the definition of the combinatorial game SIM and knows why (by virtue of the Ramsey theorem) we cannot obtain draw in this game.
- The student knows some open problems from combinatorics and she/he can formulate some consequences of solving this problems. Knowledge of some basic partition theorems (the Ramsey theorem, the Schur's theorem, the Hales-Jewett theorem, the Van der Waerden's theorem).
- Knowledge how to proof the Van der Waerden's theorem assuming the Hales-Jewett theorem.
- Knowledge of at least two open problems from combinatorics.
- The student knows the notion of a Hadamard matrix and can formulate at least one application of it.

Skills

- Dealing with examples of applications of basic combinatorial theorems for special cases.
- Ability to derive explicit formulas from recurrence formulas using standard special sequences (like Stirling numbers, Catalan numbers, Bell numbers).* Ability to compute the number of objects from a given problem/exercise using basic combinatorial theorems and notions. * Ability to compute the number of objects using the notion of combinations with repetitions.
- Ability to apply the Hall's marriage theorem to solve given problem/exercise.

	• Ability to check whether given a bipartive graph (or a system of sets) has a system of distinct representatives. • Ability to extend given a Latin rectangle to a Latin square. Also ability to prove that a not full Latin square cannot be extended to a full Latin square. • Ability to give a few nonisomorphic Latin squares of a given size. • Ability to apply the theory of fields to construction of p - 1 mutually orthogonal Latin squares of size p x p. • Ability to sketch a proof of one of the partition theorems (for example the Schur's theorem). • Ability to give an application of the equality R(3, 3) = 6. • Ability to give an interpretation of the value S(3) = 13 and an appropriate counterexample that S(3) is not below 13. • Ability to find a Hadamard matrix of size n x n for an appropriate small even n.
	Social competence

Contact

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