KAPITAŁ LUDZKI
NARODOWA STRATEGIA SPÓJNOŚCI

Projekt współfinansowany przez Unię Europejską w ramach Europejskiego Funduszu Społecznego

UNIA EUROPEJSKA
EUROPEJSKI FUNDUSZ SPOŁECZNY

Course title

Set theory

ECTS code

11.1.0332

Name of unit administrating study

null

Studies

faculty	field of study		
Faculty of Mathematics, Physics and Informatics	Mathematics	type	first tier studies (BA)
		form	full-time
		specialty	null, mathematics – teacher education
		specialization	all
Faculty of Mathematics, Physics and Informatics	Mathematics	type	second tier studies (MA)
		form	full-time
		specialty	theoretical mathematics, financial mathematics, mathematics – teacher education
		specialization	all
Faculty of Mathematics, Physics and Informatics	Mathematical Modeling and Data Analysis	type	second tier studies (MA)
		form	full-time
		specialty	all
		specialization	all

Teaching staff

prof. UG, dr hab. Andrzej Nowik; dr Paweł Klinga; prof. dr hab. Edward Grzegorek; dr hab. Rafał Filipów

Forms of classes, the realization and number of hours

Forms of classes

Auditorium classes, Lecture

The realization of activities

classroom instruction

Number of hours

Lecture: 30 hours, Auditorium classes: 30 hours

ECTS credits

5

The academic cycle

2022/2023 winter semester

Type of course

an elective course

Language of instruction

- polish
- english

Teaching methods

- problem solving
- problem-focused lecture

Form and method of assessment and basic criteria for eveluation or examination requirements

Final evaluation

- Graded credit
- Examination

Assessment methods

- (mid-term / end-term) test
- written exam with open questions
- oral exam

The basic criteria for evaluation

Method of verifying required learning outcomes

Required courses and introductory requirements

A. Formal requirements

B. Prerequisites

Aims of education

Knowledge of a notions of the set theory and its applications to topology, real functions theory and functional analysis.

Course contents

1. Axioms of ZFC set theory with explanation of their role in capturing fundamental intuistic properties of sets. Varies formulation of axioms of choice with proofs of their equivalence (e.g. existence of choice function, Zermello theorem, Kuratowski-Zorn lemma).
2. Definitions of fundamental notions of set theory with help of axioms.
3. Properties of well ordered sets. Transfinite inductions. Definitions with help of transfinite inductions.
4. Von Neumann ordinals.
5. Von Neumann cardinals.
6. Cardinal arithmetic and some of it applications to other parts of mathematics.
7. Cofinality of cardinals and Konig Theorem.
8. Natural numbers in set theory.
9. Weakly and strongly inaccesible cardinals.
10. Real and 0-1 measurable cardinals. Banach –Kuratowski Theorem. Ulam Theorem. Ulam matrix.
11. Universal measure zero sets and strong measure zero sets. Luzin set.
12. Some fundamental constructions of big family of sigma-independent sets, almost disjoint sets, cardinality of sigma-field generated by a family of sets.
13. Role of axiom of choice and continuum hypothesis in set theory.
14. Some consequences of Martin axiom for measure theory and topology.

Bibliography of literature

1. W. Guzicki, P. Zakrzewski, *Wykłady ze wstępu do matematyki. Wprowadzenie do teorii mnogości*, WN PWN Warszawa 2005.
2. P. Halmos, *Naive set theory*, Princeton 1960, Springer Verlag 1974
3. K. Kunen, *Set Theory*, North Holland, Amsterdam 1980
4. K. Kuratowski, *Wstęp do teorii mnogości i topologii*, PWN 1980.
5. K. Kuratowski, A. Mostowski, *Teoria mnogości*, PWN 1966
6. Krzysztof Ciesielski, *Set theory for working mathematician*, Cambridge University Press, 1997.
7. Aleksander Błaszczyk, Sławomir Turek, *Teoria mnogości*, PWN 2007.

The learning outcomes (for the field of study and specialization)

Knowledge

The student:

- Knows the Axioms of Zermelo-Fraenkel (with the Axiom of Choice) set theory. and their importance in the basic intuitive properties of sets. She/he knows a various formulations of the Axiom of Choice and their equivalences following from the Zermelo-Fraenkel set theory (for example the existence of a selectors, Zermelo's theorem, Zorn's lemma). She/he knowns basic notions of the set theory based on the Axioms of ZFC theory. She/he is familiar with basic properties of well ordered sets, with the technique of the transfinite induction and the transfinite recursion.
- Knows (in details) the definition of von Neumann ordinals, von Neumann cardinals, The student knows the basics of cardinal arithmetic and its applications to another parts of mathematics (for example, the construction of groups on nonempty sets with any power, size of the sigma field generated by sets of size continuum, in particular, for Borel sets on the real line). She/he is familiar with the Konig's theorem about cofinality and knows corollaries from this theorem about cofinality of the continuum and indirectly its influence on the real line).
- The student knows the interpretations of natural numbers in the set theory.
- The student if familiar with the definitions of weakly and strongly inaccessible cardinals, real-valued and 0-1 valued measurable cardinals and basic theorems about this cardinals (for example the Kuratowski-Ulam theorems, Ulam's theorems, Ulam's matrix, etc.) She/he can use this theorems to the measure theory.
- The student knows some basic constructions in the classical set theory, namely the existence of big families of sigma independent sets, pairwise almost disjoint sets, also knows the construction of universally measure zero sets strong measure zero sets, Luzin and Sierpiński sets.
- The student is familiar with some consequences of the Martin's Axiom for measure theory and topology.

M2_W01, M2_W02, M2_W03

Skills

- Ability to understand the influence of the Axioms of Zermelo-Fraenkel (with the Axiom of Choice) set theoryin deraving the basic intuitive properties of sets. Ability to prove using the Zermelo-Fraenkel set theory various forms of the

	Axiom of Choice (for example the existence of a selectors, Zermelo's theorem, Zorn's lemma). Ability to describe basic notions of the set theory based on the Axioms of ZFC theory. • Ability to use the technique of the transfinite induction and the transfinite recursion. • Ability to apply cardinal arithmetic rules to another parts of mathematics (for example, the construction of groups on nonempty sets with any power, size of the sigma field generated by sets of size continuum, in particular, for Borel sets on the real line). • Ability to understand the proof of the Konig's theorem about cofinality and ability to prove using this theorem a corollaries about cofinality of the continuum and indirectly about the real line. • Ability to use the notion of natural numbers in the set theory and to use the notions of weakly and strongly inaccessible cardinals, real-valued and 0-1 valued measurable cardinals and ability to understand basic theorems about this cardinals (for example the Kuratowski-Ulam theorems, Ulam's theorems, Ulam's matrix, et cetera). • Ability to understand some basic constructions in the classical set theory, namely the existence of big families of sigma independent sets, pairwise almost disjoint sets, also the construction of universally measure zero sets strong measure zero sets, Luzin and Sierpiński sets, et cetera. • Ability to apply some consequences of the Martin's Axiom to measure theory and topology. M2_U01, M2_U03, M2_U04, M2_U05, M2_U06, M2_U07
	Social competence

Contact

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